Math 53: Multivariable Calculus **Sections 201, 204**

Worksheet for 2021-09-08

Conceptual questions

Question 1. See the back of this handout.

Computations

First, some questions on Chapter 12.

Problem 1. Are there any vectors $\mathbf{v}$ such that $\langle 1,2,1\rangle \times \mathbf{v} = \langle 3,1,-5\rangle$? If so, find all of them. If not, explain why not.

Then do the same question, but for $\langle 1,2,1\rangle \times \mathbf{v} = \langle 3,1,5\rangle$.

Problem 2. Let L_1 be the line passing through $A(1,-2,4)$ and $B(2,1,3)$, and let L_2 be the line passing through $C(0,3,-3)$ and $D(2,4,1)$.

Are L_1, L_2 parallel, skew, or intersecting? If they intersect, where do they intersect? If not, how far apart are they?

The next ones introduce some §13.2 content into the mix.

Problem 3. Find the point where the curves $\mathbf{r}_1(t) = \langle 2t, 2-2t, 3+t^2\rangle$ and $\mathbf{r}_2(t) = \langle 6-2t, 2t-4, t^2\rangle$ intersect. Then compute the angle formed by the two curves at their point of intersection.

Problem 4. Let $\mathbf{r}_1(t)$ be as in the preceding problem, and let H be the plane $x+y-2z+6=0$. The curve given by $\mathbf{r}_1$ intersects H twice. Find these two points, and determine the angle at which the curve meets the plane at each point.

The following are solutions to the problem

"Find the distance d between the point $P(1,-2,2)$ and the line $\mathbf{r}(t) = \langle 3+3t, 2-t, 5t\rangle$."

Figure out what is happening in each one.

(a) Solution 1:

$$\begin{aligned} D^2 &= (2+3t)^2 + (4-t)^2 + (5t-2)^2 \\ &= 35t^2 - 16t + 24 \\ \frac{\mathrm{d}}{\mathrm{d}t}(D^2) &= 70t - 16 = 0 \\ t &= 8/35 \\ d &= D_{\min} = \sqrt{35(8/35)^2 - 16(8/35) + 24} = 2\sqrt{194/35}. \end{aligned}$$

(b) Solution 2:

$$\begin{aligned} 3(x-1) - (y+2) + 5(z-2) &= 0 \\ 3x - y + 5z - 15 &= 0 \\ 3(3+3t) - (2-t) + 5(5t) - 15 &= 0 \\ t &= 8/35 \\ d &= \sqrt{(3+3(8/35)-1)^2 + (2-(8/35)+2)^2 + (5(8/35)-2)^2} = 2\sqrt{194/35}. \end{aligned}$$

(c) Solution 3:

$$\begin{aligned} \langle 1,-2,2\rangle - \langle 3,2,0\rangle &= \langle -2,-4,2\rangle \\ \langle -2,-4,2\rangle \times \langle 3,-1,5\rangle &= \langle -18,16,14\rangle \\ |\langle -18,16,14\rangle| &= 2\sqrt{194} \\ |\langle 3,-1,5\rangle| &= \sqrt{35} \\ d &= 2\sqrt{194/35}. \end{aligned}$$

(d) Solution 4:

$$\begin{aligned} \langle 1,-2,2\rangle - \langle 3,2,0\rangle &= \langle -2,-4,2\rangle \\ (\langle 3,-1,5\rangle \times \langle -2,-4,2\rangle) \times \langle 3,-1,5\rangle &= \langle 94,132,-30\rangle = 2\langle 47,66,-15\rangle \\ \frac{\langle 47,66,-15\rangle \cdot \langle -2,-4,2\rangle}{|\langle 47,66,-15\rangle|} &= -388/\sqrt{6790} \\ d &= |-388/\sqrt{6790}| = 2\sqrt{194/35}. \end{aligned}$$

(e) Solution 5:

$$\begin{aligned} \langle 2+3t, 4-t, 5t-2\rangle \cdot \langle 3,-1,5\rangle &= 0 \\ 35t - 8 &= 0 \\ t &= 8/35 \\ d &= \sqrt{(3+3(8/35)-1)^2 + (2-(8/35)+2)^2 + (5(8/35)-2)^2} = 2\sqrt{194/35} \end{aligned}$$

(f) Solution 6:

$$\begin{aligned} \langle 1,-2,2\rangle - \langle 3,2,0\rangle &= \langle -2,-4,2\rangle \\ \frac{\langle 3,-1,5\rangle \cdot \langle -2,-4,2\rangle}{\langle 3,-1,5\rangle \cdot \langle 3,-1,5\rangle}\langle 3,-1,5\rangle &= \frac{8}{35}\langle 3,-1,5\rangle \\ \langle -2,-4,2\rangle - \frac{8}{35}\langle 3,-1,5\rangle &= \langle -\frac{94}{35}, -\frac{132}{35}, \frac{6}{7}\rangle \\ |\langle -\frac{94}{35}, -\frac{132}{35}, \frac{6}{7}\rangle| &= 2\sqrt{194/35}. \end{aligned}$$